$A \to B \to C$

$A \leftarrow B \leftarrow C$

Causal chain

$A \to B \leftarrow C$ $\to$ collider / fork / V-struc

common effect

$A \leftarrow B \to C$

Common cause

$$A \to B \to C$$

$$A \to B \to C$$

Shaded nodes $\Rightarrow$ observed/given

$X = \{A\}$

$Y = \{C\}$

$E = \{\}$

$X \perp\!\!\!\perp Y \mid E$? not guaranteed.

$P(A, C \mid b)$

$= P(A \mid b)\, P(C \mid b)$

$X = \{A\}$

$Y = \{C\}$

$E = \{B\}$

$X \perp Y \mid E \rightarrow$ guaranteed.

$$A \rightarrow B \rightarrow C$$

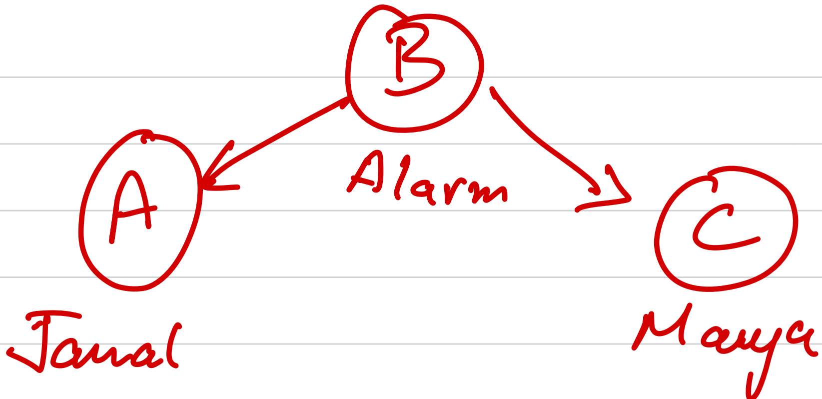
$$\longrightarrow \ \| \ \longrightarrow$$

$$P(A,C|b) = \frac{P(A,C,b)}{P(b)}$$

$$= \frac{P(A) \cdot P(b|A) \cdot P(C|b)}{P(b)}$$

Bayes rule

$$P(A|b) \cdot P(C|b)$$



$$P(A,C|b)$$

$$= \frac{P(A,C,b)}{P(b)}$$

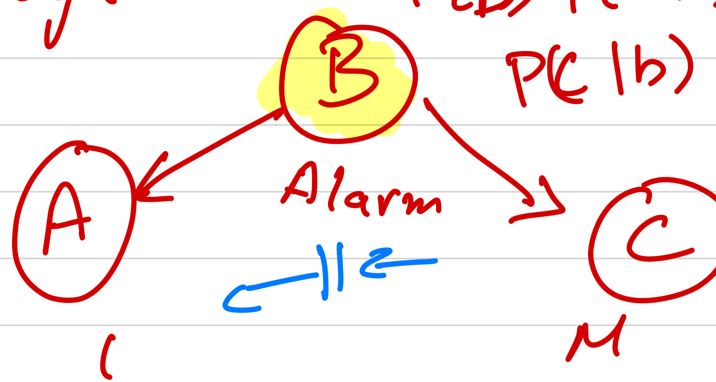
$$P(B)\cdot P(A|b)$$

$$P(C|b)$$

$X = \{A\}$ $Y = \{C\}$

$E = \{\ \}$

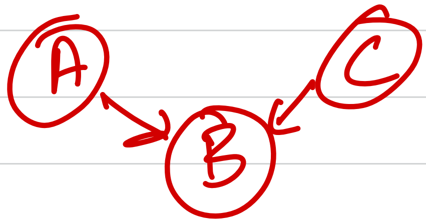
$X \perp\!\!\!\perp Y \mid E$ → not guaranteed.



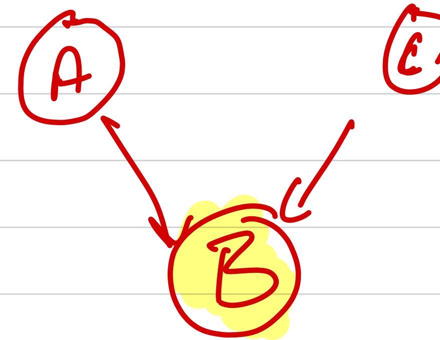
$X = \{A\}$

$Y = \{C\}$

$E = \{B\}$



$A \perp C \mid \{\}$

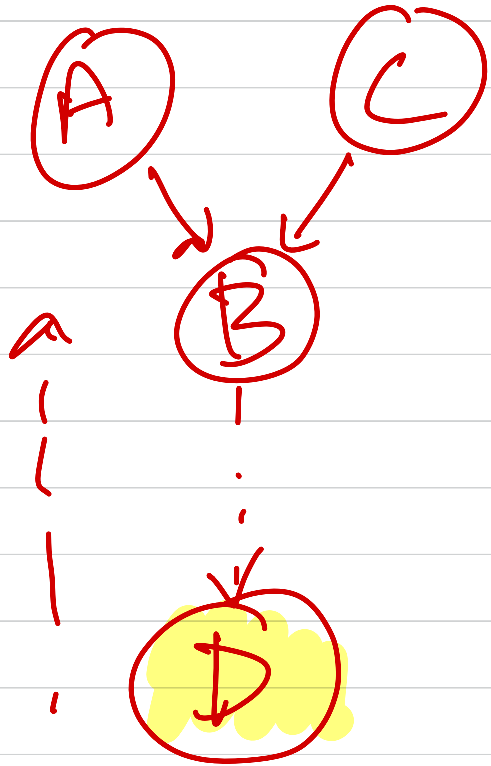


$A \not\perp C \mid B$

$$P(A,C) = P(A)\cdot P(C)$$

$$\sum_B P(A,C,B) \Rightarrow \sum_B P(A)\cdot P(B|A,C)\cdot P(C)$$

$$= \underbrace{P(A)\cdot P(C)} \sum_B P(B|A,C) \rightarrow 1$$


A
C
B
D